Response of Dense Relativistic Matter to A Magnetic Field

Volodya Miransky

E.V.Gorbar, V.M., and I. Shovkovy, Phys. Rev. C80 (2009), 032801(R)
+ work in progress

Dense relativistic matter

- Dense relativistic matter is common inside compact stars
  - Electrons in white dwarfs

    $\mathrm{T} \ll \mathrm{m} \lesssim \mu$ (*i.e.*, T$\lesssim$ 1 keV & $\mu \simeq 1$ MeV)
  - Neutrons of nuclear matter

    $\mathrm{T} \ll \mathrm{m} \lesssim \mu$ (*i.e.*, T$\lesssim$ 10 MeV & $\mu \simeq 1$ GeV)
  - Electrons inside stellar nuclear matter

    $\mathrm{m} \lesssim \mathrm{T} \ll \mu$ (*i.e.*, T$\lesssim$ 10 MeV & $\mu \simeq 100$ MeV)
  - Dense quark matter in stellar cores (if formed)

    $\mathrm{T} \lesssim \mathrm{m} \ll \mu$ (*i.e.*, T$\lesssim$ 10 MeV & $\mu \gtrsim 400$ MeV)

Zero Density State

- Magnetic catalysis (dynamical generation of a nonzero Dirac mass even at $g \ll 1$)

[V. Gusynin, V.M., I. Shovkovy, PRL 73, 3499 (1994); PLB 349, 477]

PHYSICAL REVIEW D **66**, 045006 (2002)

Magnetic catalysis and anisotropic confinement in QCD

V. A. Miransky*
Department of Applied Mathematics, University of Western Ontario, London, Ontario, Canada N6A 5B7

I. A. Shovkovy*
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

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The expressions for dynamical masses of quarks in the chiral limit in QCD in a strong magnetic field are obtained. A low energy effective action for the corresponding Nambu-Goldstone bosons is derived and the values of their decay constants as well as the velocities are calculated. The existence of a threshold value of the number of colors N_c^{thr}, dividing the theories with essentially different dynamics, is established. For the number of colors $N_c \ll N_c^{thr}$, an anisotropic dynamics of confinement with the confinement scale much less than Λ_{QCD} and a rich spectrum of light glueballs is realized. For N_c of order N_c^{thr} or larger, a conventional confinement dynamics takes place. It is found that the threshold value N_c^{thr} grows rapidly with the magnetic field [$N_c^{thr} \gtrsim 100$ for $|eB| \gtrsim (1\ \mathrm{GeV})^2$]. In contrast with QCD with a nonzero baryon density, there are no principal obstacles for examining these results and predictions in lattice computer simulations.

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A. J. Mizher, M. N. Chernodub and E. S. Fraga, *Phase diagram of hot QCD in an external magnetic field: possible splitting of deconfinement and chiral transitions*, arXiv:1004.2712 [hep-ph].

General idea

- Topological current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle \bar{\psi}\gamma^3\gamma^5\psi\rangle = \frac{eB}{2\pi^2}\mu$$ (free theory!)

[Metlitski, Zhitnitsky, PRD **72**, 045011 (2005)]

- Should there be a dynamical parameter Δ, associated with an axial-vector condensate $\langle A_5^3 \rangle$?

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta \quad \text{where} \quad \boxed{\mathcal{L}_\Delta \simeq \Delta\bar{\psi}\gamma^3\gamma^5\psi}$$

- Note: Δ=0 is not protected by any symmetry

Lesson from graphene

- Dynamics of Quantum Hall Effect in graphene

  (≈ 2-brane QED)

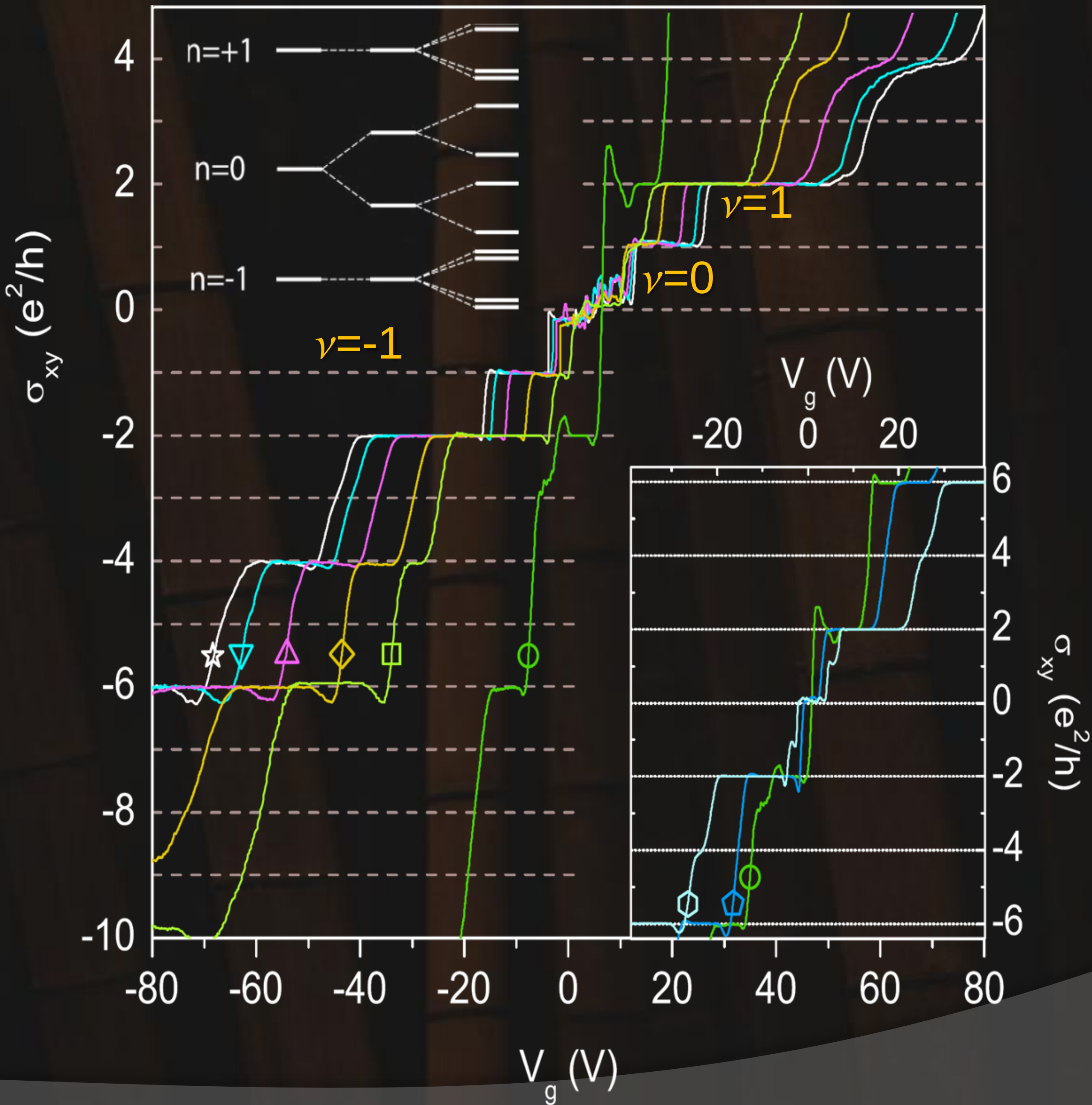
  - Parity and time-reversal odd Dirac (Chern-Simons) mass*

$$\Delta \quad \sim \quad \langle \bar{\Psi} \gamma^3 \gamma^5 \Psi \rangle$$

- Δ describes the 0^{th} plateau in Quantum Hall effect in graphene

$$iG^{-1}(u,u') \;=\; \Big[(i\partial_t + \mu)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3 + \; i\tilde{\mu}\gamma^1\gamma^2 + \Delta\gamma^3\gamma^5 - m\Big]\delta^4(u-u')$$

*[Gorbar, Gusynin, V.M., Shovkovy., PRB 78, 085437 (2008)]



Zhang et al.,
PRL **96,** 136806 (2006)

Model

- Lagrangian density:

$$\mathcal{L} = \bar{\psi}\left(iD_\nu + \mu_0 \delta_\nu^0\right)\gamma^\nu \psi + \frac{G_{\mathrm{int}}}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right]$$

- The dimensionless coupling is

$$g \equiv G_{\mathrm{int}}\Lambda^2/(4\pi^2) \ll 1$$

- Magnetic field is inside $D_\nu = \partial_\nu - ieA_\nu$
  where $A_\nu = xB\delta_\nu^2$ (Landau gauge)

Approximation

- Gap equation in mean-field approximation:

$$G^{-1}(u,u') = S^{-1}(u,u') - iG_{\mathrm{int}}\left\{G(u,u) - \gamma^5 G(u,u)\gamma^5 - \mathrm{tr}[G(u,u)] + \gamma^5\,\mathrm{tr}[\gamma^5 G(u,u)]\right\}\delta^4(u-u')$$

where

$$iG^{-1}(u,u') = \Big[(i\partial_t + \mu)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3 + i\tilde{\mu}\gamma^1\gamma^2 \boxed{+\,\Delta\gamma^3\gamma^5} - m\Big]\delta^4(u-u')$$

and

$$iS^{-1}(u,u') = \left[(i\partial_t + \mu_0)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3\right]\delta^4(u-u')$$

Vacuum state

- Magnetic catalysis (spontaneous generation of a nonzero Dirac mass even at $g \ll 1$):

$$m_{\mathrm{dyn}}^2 = \frac{1}{\pi l^2} \exp\left(-\frac{\Lambda^2 l^2}{g}\right) \text{ where } l = 1/\sqrt{|eB|}$$

  (along with $\mu = \mu_0$)

  [Gusynin, V.M., Shovkovy, PRL **73**, 3499 (1994); PLB **349**, 477 (1995)]

- The solution exists for $\mu_0 < m_{dyn}$, although it will be less stable than the normal state ($m = 0$) already for $\mu_0 \gtrsim m_{\mathrm{dyn}}/\sqrt{2}$ [Clogston, PRL **9**, 266 (1962)]

“Abnormal” normal ground state

- The gap equation allows another solution,

$$\mu \simeq \mu_0 \text{ and } \Delta \simeq g\mu_0 eB/\Lambda^2$$

- This solution is almost independent of temperature when $T \ll \mu$
- This is the normal ground state since its symmetry is same as in the Lagrangian
- Besides, there is no trivial solution Δ=0

Change of ground state

- The free energy in the state with $m \neq 0$ (broken symmetry)

$$\Omega_m \simeq -\frac{m_{\rm dyn}^2}{2(2\pi l)^2}\left(1 + (m_{\rm dyn} l)^2 \ln|\Lambda l|\right)$$

- The free energy in the normal state, $\Delta \neq 0$

$$\Omega_\Delta \simeq -\frac{\mu_0^2}{(2\pi l)^2}\left(1 - g\frac{|eB|}{\Lambda^2}\right)$$

- So, indeed symmetry is restored for $\mu > \mu_c$,

$$\mu_c \simeq m_{\rm dyn}/\sqrt{2}$$

Physical meaning of Δ

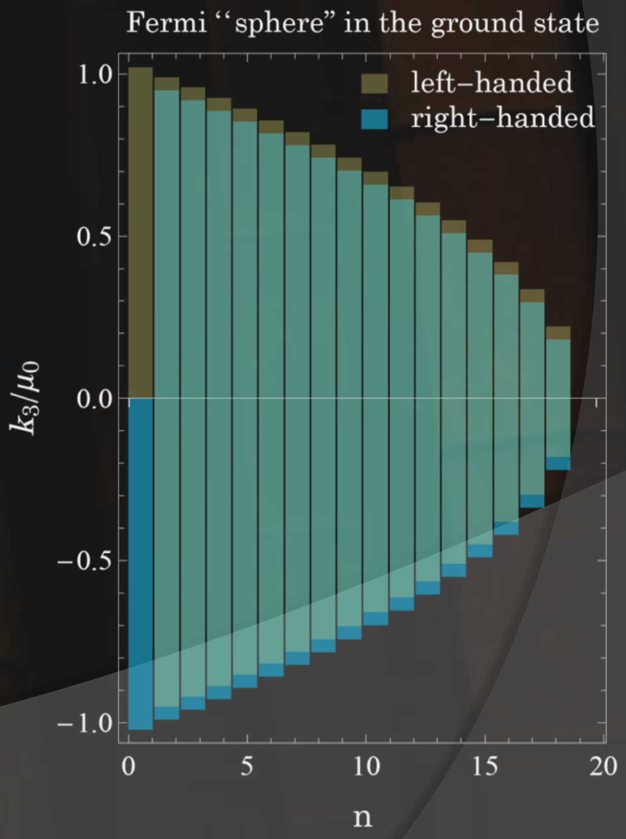
- The dispersion relation of quasiparticles:

$$\omega_{n,\sigma} = -\mu \pm \sqrt{[k_3 + \sigma\Delta]^2 + 2n|eB|}$$

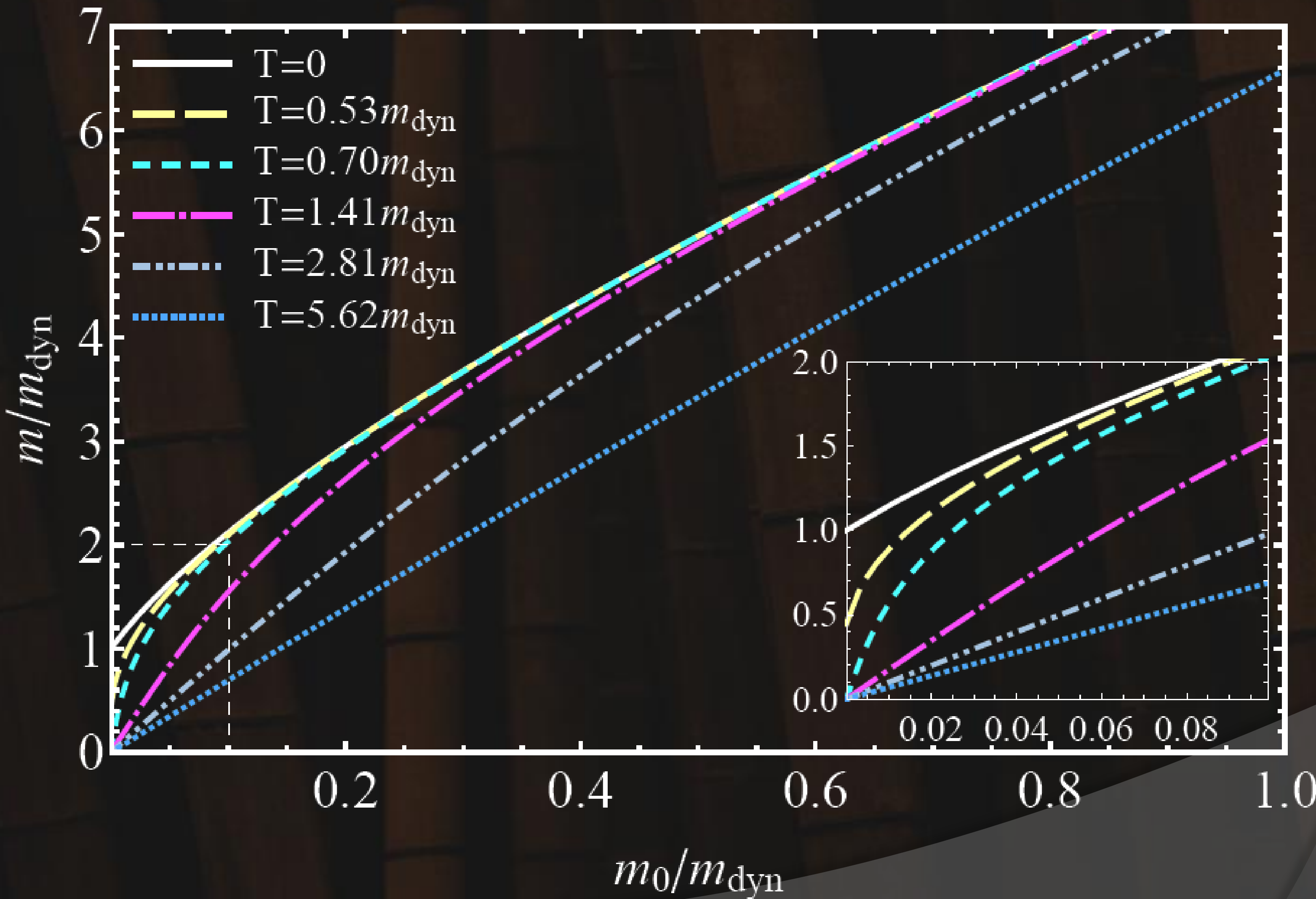
where $\sigma = \pm 1$ is the chirality

- Longitudinal momenta of opposite chirality fermions are *shifted*, *i.e.*, $k_3 \to k_3 \pm \Delta$

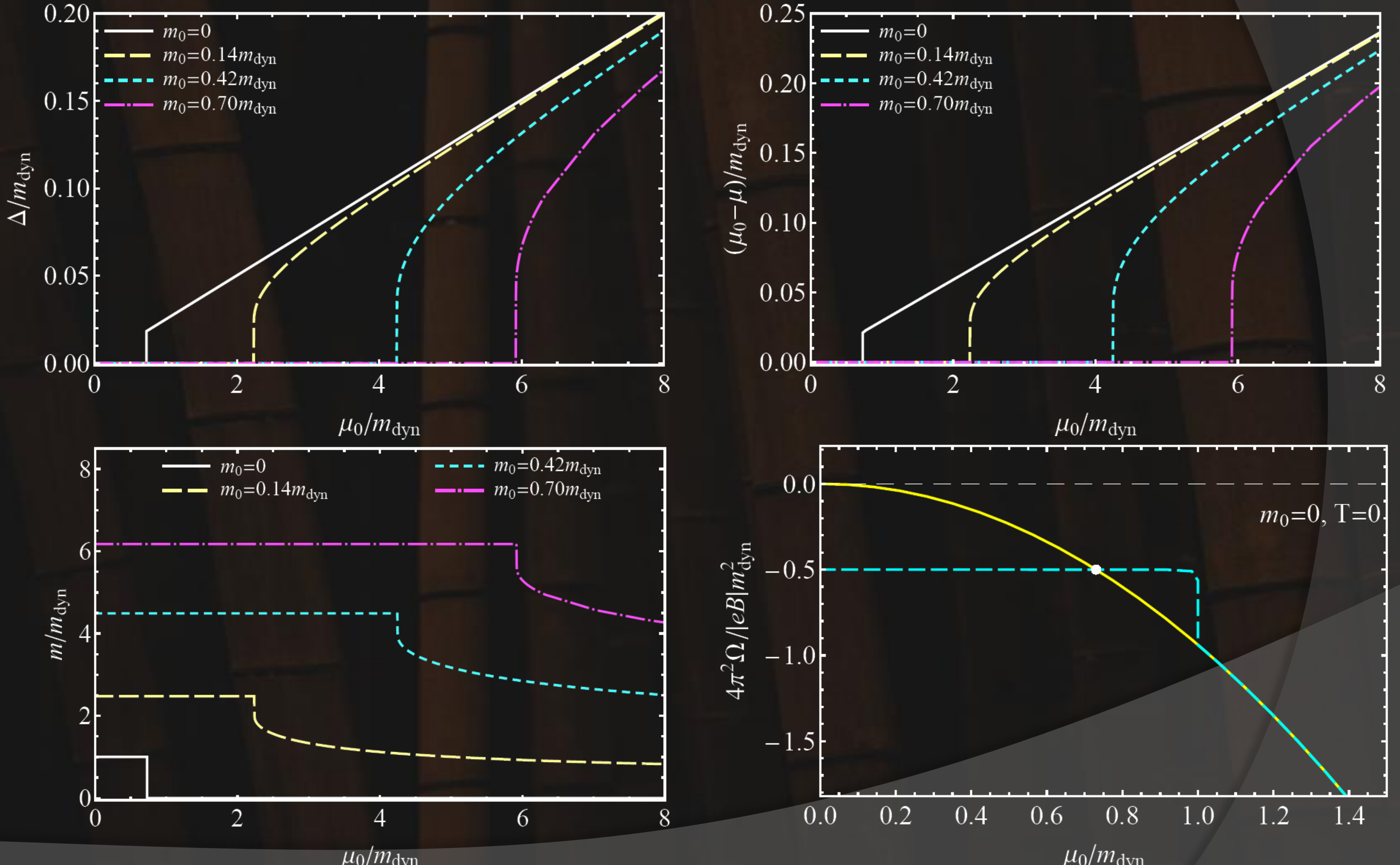
- All Landau levels ($n \geq 0$) are affected by Δ



Magnetic catalysis at μ_0=0

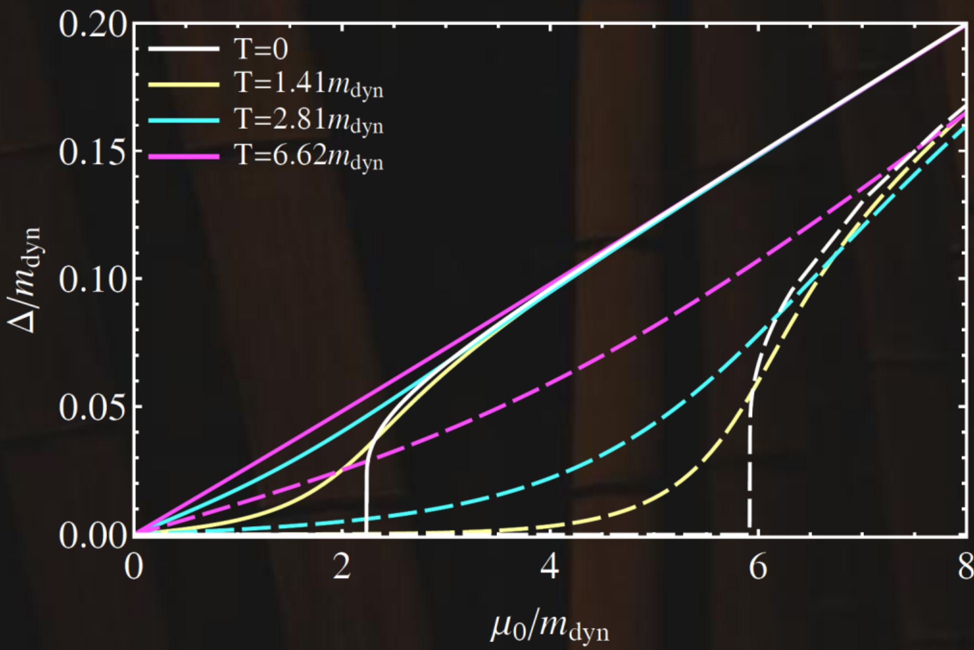


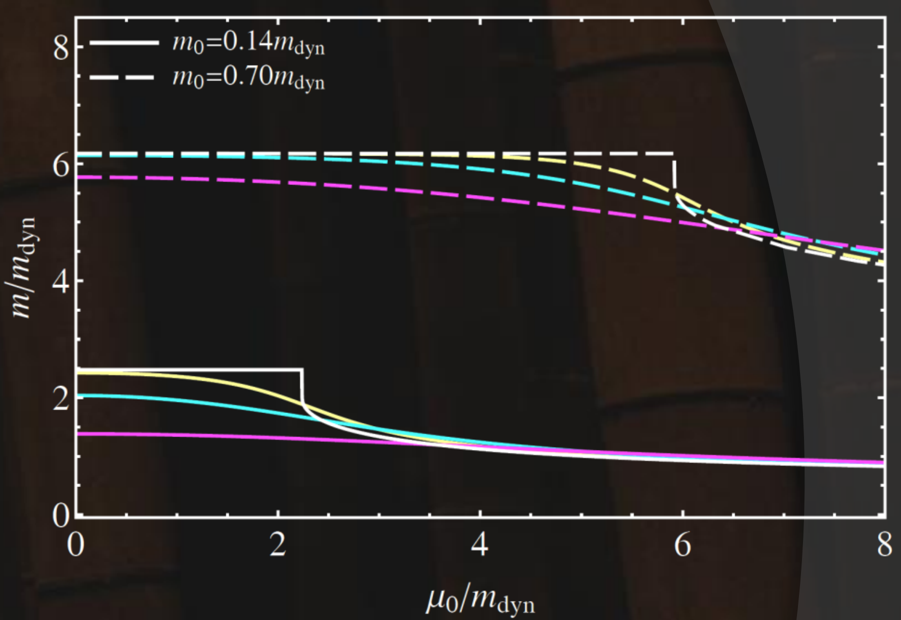
T=0 results



m_0=0
m_0=0.14m_{dyn}
m_0=0.42m_{dyn}
m_0=0.70m_{dyn}
Δ/m_{dyn}
$(\mu_0-\mu)/m_{dyn}$
m/m_{dyn}
$4\pi^2\Omega/|eB|m_{dyn}^2$
m_0=0, T=0
μ_0/m_{dyn}

T≠0 results





- These are smoothed versions of the T=0 results
- The dependence μ-μ_0 versus μ_0 (not shown) at T≠0 is similar to Δ versus μ_0 (shown)

Induced axial current

- The axial current in the ground state is

$$\langle \bar{\psi}\gamma^3\gamma^5\psi\rangle \;=\; \boxed{\frac{eB}{2\pi^2}\mu} - \frac{|eB|}{2\pi^2}\Delta - \frac{|eB|}{\pi^2}\Delta\sum_{n=1}^{\infty}\kappa(\sqrt{2n|eB|},\Lambda)$$

- In addition to the topological contribution, $\frac{eB}{2\pi^2}\mu$ there are dynamical ones $\propto \Delta$
- An equivalent result is also obtained in the Pauli-Villars regularization
- Note: on the solution to the gap equation:

$$\langle j_5^3(u)\rangle = \frac{2\Delta}{G_{\text{int}}} = \frac{\Delta}{2\pi^2}\frac{\Lambda^2}{g}$$

Potential implications

- Physical properties to be affected
  - transport
  - emission

    (must be sensitive to anisotropy and/or CP violation)
- Specific physical systems
  - Compact stars
    - Quark stars (quarks)
    - Hybrid stars (quarks, electrons)
    - Neutron stars (electrons)
    - White dwarfs (electrons)
  - Heavy ion collisions [Kharzeev & Zhitnitsky, NPA **797**, 67(2007), Kharzeev, McLerran & Warringa, NPA 803, 227 (2008), …]

Pulsar kicks

- The dynamical chiral shift parameter is driven by chemical potential ($T \ll \mu$)

$$\Delta \simeq g\mu_0 eB/\Lambda^2$$

  and is almost independent of temperature

- This creates an anisotropy in the distribution of left-handed quarks/electrons
- The anisotropy is transferred to left-handed neutrinos by elastic scattering
- Pulsar gets a kick when neutrinos escape

Supernova explosions

- Because of the robustness of the axial currents at finite temperatures, even supernova explosions may be affected

- A small early-time neutrino asymmetry may *facilitate* explosions and give a kick at the same time, e.g., see

  [Fryer & Kusenko, Astrophys. J. Supp. **163**, 335 (2006)]

Recent related works

- A. Rebhan, A. Schmitt and S. A. Stricker, *Anomalies and the chiral magnetic effect in the Sakai-Sugimoto model*, JHEP 1001, 026 (2010)
- G. Basar, G. V. Dunne and D. E. Kharzeev, *Chiral Magnetic Spiral*, arXiv:1003.3464 [hep-ph].
- K. Fukushima and M. Ruggieri, *Dielectric correction to the Chiral Magnetic Effect*, arXiv:1004.2769 [hep-ph].
  (Modification of the CME due to “vector-like” Δ)

Summary

- Main message: Order parameters in relativistic systems with a non-Lorentz invariant ground state can be quite different from conventional Lorentz invariant order parameters
- $\mu < \mu_c$: Chiral symmetry is broken in the ground state (magnetic catalysis)
- $\mu > \mu_c$: Normal ground state of dense relativistic matter in a magnetic field is characterized by
  - Chiral shift parameter (may have dramatic implications for stars)
  - Axial current along the field (physical effects are not obvious)
  - No solution with vanishing Δ exists

Outlook

- Calculation of neutrino emission/diffusion in the state with a chiral shift parameter (work in progress)
- Transport properties of the normal state with nonzero chiral shift parameter
- The fate of the induced axial current in the renormalized models (work in progress)

Thank you